Viscous fingering in complex fluids

Anke Lindner, Daniel Bonn and Jacques Meunier

Laboratoire de Physique Statistique, École Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cédex 05, France

Abstract.　Viscous fingers form when a less viscous fluid pushes a more viscous fluid in a linear channel. The instability of the interface results from a competition between viscous and capillary forces. We show here that by using complex fluids such as polymer or surfactant solutions one can act on the viscosity or the surface tension and modify the instability drastically. Two different polymer solutions, that exhibit either shear thinning or normal stress effects, are used. For the first fluid the viscous forces are altered leading to finger narrowing, whereas for the second fluid the viscous forces are supplemented by normal stresses, which leads to finger widening. For the surfactant solutions the modification of the capillary forces leads to finger widening.

1. Introduction

The Saffman–Taylor instability [1, 2] arises when a less viscous fluid pushes a more viscous fluid in a thin linear channel or Hele–Shaw cell. The interface between the two fluids develops an instability, leading to the formation of fingerlike patterns called viscous fingers. Because of its relative simplicity it has received much attention [3] and is considered as an archetype of pattern forming systems. Moreover, it is considered as a model system for flow in porous media and a limiting factor in the recovery of crude oil.

In a rectangular channel, one usually quantifies the width of the finger w relative to the channel width W, $\lambda = w/W$ as a function of the finger velocity v. It is determined by the capillary number $\mathrm{Ca} = \Delta\mu v/\sigma$, which represents the ratio of viscous forces to capillary forces: $\Delta\mu$ is the viscosity difference between the two fluids and σ the surface tension. The viscous forces tend to narrow the fingers whereas the capillary forces tend to widen the finger. As a result λ decreases with increasing finger velocity. For large values of the capillary number, λ reaches a limiting value of $1/2$ [1]. The control parameter is $1/B = 12\mathrm{Ca}(W/b)^2$, where b is the distance between the plates of the Hele–Shaw cell. When scaled on $1/B$, measurements of λ for different systems all fall on the same universal curve.

Lately the instability has also been studied for complex fluids [4, 5]. Here we consider two different dilute polymer solutions, that each exhibit only one non-Newtonian property, in the sense that other non-Newtonian effects can be neglected. We investigate the instability for solutions of a rigid polymer that exhibit a shear thinning viscosity and for solutions of a flexible polymer that exhibit mainly elastic effects i.e. normal stress effects. This allows for separate investigation of the influence of the two non-Newtonian properties on the Saffman–Taylor instability. In addition a surfactant solution has been investigated.

We can thus modify the selection mechanism of the finger in different ways. For the rigid polymer, the viscous forces are modified, for the flexible polymer they are supplemented by

another (dominant) force and for the surfactant solution the capillary forces are modified. The use of these fluids allows one to tune the Saffman–Taylor instability in a specific way.

2. Experiment

The viscous fingering experiments are performed in a rectangular Hele–Shaw cell consisting of two glass plates separated by a thin Mylar spacer, fixing the plate spacing $b = 0.25$ mm. The cell is filled with the polymer or surfactant solution. For the polymer solutions compressed air is used as the less viscous driving fluid; the surfactant solution is driven by an oil of low viscosity (heptane). The fingers are captured by a CCD camera coupled to a VCR. This allows for measurements of their width w as a function of the velocity v.

The two different polymer solutions are first a solution of the stiff rodlike polymer Xanthane with an average molecular weight of $M = 3 \times 10^6$, and secondly a solution of the flexible polymer polyox (PEO) WSR301 with a molecular weight of $M = 4 \times 10^6$.

AOT (sodium bis (ethylhexyl) sulphosuccinate) is used as the surfactant.

2.1. Polymer solutions

Polymer solutions exhibit a variety of non-Newtonian effects, such as, for instance, elastic effects such as normal stresses and a shear rate dependence of the viscosity [6]. These effects can be more or less important, depending on the nature of the polymers.

The non-Newtonian viscosities as well as the normal stress differences of the polymer solutions are measured on a Reologica Stress-Tech rheometer (cone plate geometry), that is equipped with a normal force transducer. Note that neither of the polymers alters the interfacial tension much [7, 8]. The predominant effect is due to the non-Newtonian behaviour of these fluids.

Solutions of the stiff polymer Xanthane show a strong shear rate dependence of the viscosity, but there are no measurable normal stresses for the concentrations used in our experiments [9]. The viscosity decreases with increasing shear rate: the fluid is said to be shear thinning. Changing the concentration allows us to go from weak shear thinning for low concentrations to strong shear thinning for high concentrations [8].

The flexible polymer PEO, in contrast, exhibits strong elastic effects whilst its viscosity is nearly independent of the shear rate [9]. Normal stresses are caused by the stretching of the flexible polymers by the flow. The tendency of the polymer to come back to its equilibrium shape results in a stress difference between the direction of the flow and the directions perpendicular to it.

For the shear thinning solutions of Xanthane the relative finger width has been measured as a function of the finger velocity for different polymer concentrations [8, 9]. To compare the results for Newtonian fluids λ is scaled on the control parameter $1/B$ (figure 1). The non-Newtonian viscosity has been taken into account by replacing μ by the shear dependent viscosity $\mu(\dot{\gamma})$, where the shear rate in the cell can be estimated as $\dot{\gamma} \simeq v/b$.

We find that for low polymer concentrations the finger width as a function of $1/B$ falls on the universal curve for Newtonian fluids, given by the numerical results of McLean and Saffman [2]. For increasing polymer concentration the results for the finger width start to deviate from the universal curve and values of λ smaller than $1/2$ are found for large $1/B$. The fingers become narrower as the shear thinning behaviour becomes more pronounced.

From experiments with Newtonian fluids it is well known that adding an anisotropy to the system, for example by introducing a bubble in front of the finger, causes the fingers to narrow [10]. A similar effect is likely to cause the finger narrowing also for the polymer solution. In
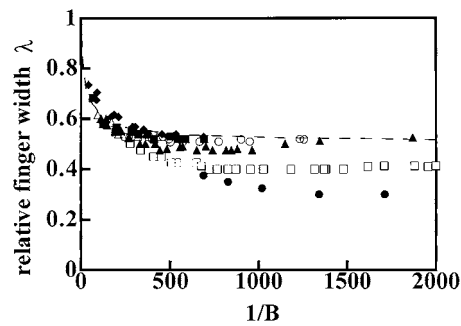
Figure 1. Relative finger width as a function of $1/B$ for water (◆) and solutions of Xanthane of 50 ppm (△), 100 ppm (■), 200 ppm (○), 500 ppm (▲), 1000 ppm (□), 1750 ppm (●); (- - - -), theoretical predications for Newtonian fluid of McLean and Saffman.
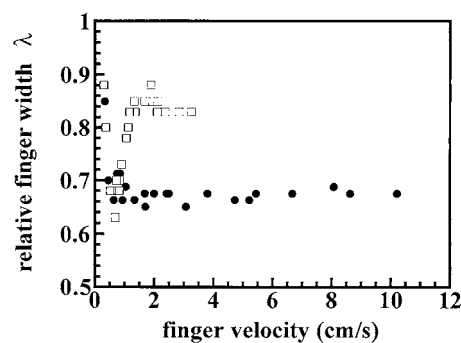


Figure 2. Relative finger width as a function of v for solutions of PEO of 60 ppm (●) and 100 ppm (□).

the region in front of the finger tip fluid velocity is high corresponding to high shear rates and, due to the shear thinning character of the fluid to small viscosities [11]. This leads to a preferred growth direction in the direction of movement of the finger. However we observe a smooth transition from the classical result to smaller fingers; experiments for Newtonian fluids have shown that introducing even a small anisotropy can lead to a strong decrease of the finger width.

A theoretical study of the finger width for a shear thinning fluid, that has been performed by Corvera Poiré and Ben Amar [12], also indicates that for high v fingers exist that are narrower than the classical result.

When measuring the relative finger width for solutions of the flexible polymer PEO which exhibit normal stress effects strikingly different results from the shear thinning solutions of Xanthane are found [7, 9, 13]: for solutions of PEO λ tends towards plateau values that are significantly higher than the classical limit 1/2 for the large velocities (figure 2).

In order to uncover the mechanism responsible for the finger widening a geometry simpler than that of the fingers in a Hele–Shaw cell was studied: the shape of moving long (but finite) air bubbles in a capillary tube: the so-called Bretherton problem [14]. In the experiment [9] the thickness of the liquid film between the bubbles and the inner wall of the capillary tube is measured, as a function of the bubble velocity (figure 3). For the very dilute PEO solutions having the same shear velocity as water, the film thickness is found to be about ten times larger than for water. The shear, which is predominantly in the plane perpendicular to the bubble surface, causes the normal stress difference to exert an extra pressure on the bubble which
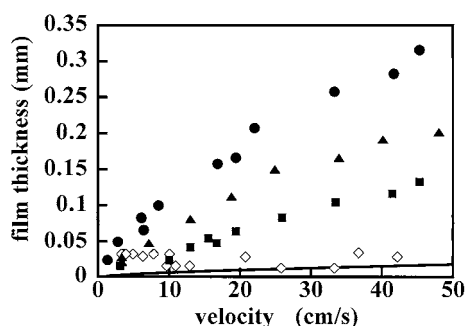
Figure 3. Thickness of the film between the bubble and the wall as a function of the bubble velocity: ◇, results for water, ——, prediction of the Bretherton theory, results for the solutions of PEO of 5 ppm (■), 50 ppm (▲) and 500 ppm (●).

leads to a thickening of the liquid film between the bubble and the capillary wall and tends to narrow the bubble.

The normal stress effects being much stronger than the viscous forces, the latter can to a first approximation be neglected and the shape of the bubble now results from the equilibrium between the capillary forces and the normal stress difference. The normal stress difference can then be estimated from the film and compares well to rheological measurements [9].

Furthermore a study of finite bubbles in the Hele–Shaw cell has been performed. At the same velocity the bubble width and the finger width are found to be identical. This supports the idea that, as was observed for the circular bubbles, the normal stress could cause thickening of the thin wetting film between the viscous finger and the glass plates of the Hele–Shaw cell. This extra pressure exerted on the finger could account for the finger widening.

2.2. Surfactant solutions

For surfactant solutions, the interface between the two fluids is completely covered by a monomolecular film of surfactant, lowering the surface tension. A low-viscosity oil (heptane) pushes a dilute solution of the surfactant AOT. The results of the finger width as a function of the finger velocity are shown in figure 4. We observe once again very wide fingers, reaching a plateau value that is well above the classical limit $\lambda = 0.5$ for high velocities.

In the viscous fingering experiment the tangential hydrodynamic flow sweeps away the surfactant film from the tip to the back of the finger whereas the interface is supplied with fresh surfactant from the bulk. The result is an anisotropic surface tension which decreases from the tip to the back of the finger and induces a widening of the finger. Similar results are found for Newtonian fluids, when an adverse anisotropy is added to the system [5].

Using a simple model for the dynamic surface tension Ben Amar performed a complete numerical resolution of the non-linear problem to account for the finger width [7, 13]. In this way a satisfactory fit of the data could be obtained (see figure 4).

3. Conclusion

We have investigated the Saffman–Taylor instability for three different complex fluids: a solution of the rigid polymer Xanthane, a solution of the flexible polymer PEO and a solution of the surfactant AOT. For each of these solutions the classical selection mechanism is altered in a different way: for the solution of Xanthane the viscous forces are modified, for the PEO
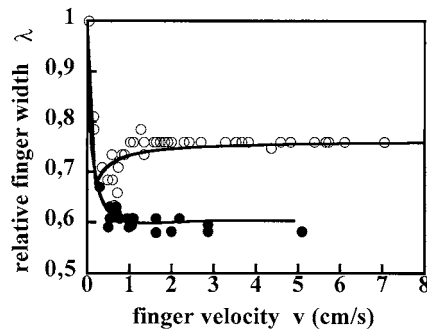
Figure 4. Relative finger width as a function of the finger velocity: ——, results of the numerical calculations, results of the solutions of AOT of 3 mM (●) and 15 mM (○).

solution they are supplemented by normal stresses; finally the surfactant acts on the surface tension.

The solutions of Xanthane exhibit a shear thinning viscosity. One finds that for weak shear thinning it is sufficient to replace μ by $\mu(\dot\gamma)$ in the control parameter $1/B$ to retrieve the classical results for the finger width. For stronger shear thinning deviations towards narrower fingers are found at high velocities, the narrowing becoming more important as the shear thinning becomes more pronounced.

For the solutions of the flexible polymer PEO normal stress effects are present and dominate the viscous forces. From experiments on bubbles in a circular geometry one knows that normal stresses can cause the thickness of the film between the bubble and the wall to increase. A similar effect is likely to increase the thickness of the thin wetting film between the finger and the glass plates leading to a widening of the fingers.

For the surfactant solutions in equilibrium the interface between the two fluids is completely covered by a film of surfactants. During finger propagation the hydrodynamic flow sweeps away the surfactant film from the tip to the back of the finger whereas fresh surfactant from the bulk is incorporated in the interface. The result is a dynamic and anisotropic surface tension which decreases from the tip to the back of the finger, and induces a widening of the finger.

In conclusion we have shown that when using complex fluids for the Saffman–Taylor instability one can alter the selection mechanism in specific ways and thus tune the instability. This may in addition, also become a way to learn something about the complex fluids themselves.

Acknowledgments

References

[1] Saffman P G and Taylor G I 1958 *Proc. R. Soc.* A **245** 312
[2] McLean J W and Saffman P G 1981 *J. Fluid Mech.* **102** 455
[3] Bensimon D, Kadanoff L P, Liang S, Shraiman B I and Tang C 1986 *Rev. Mod. Phys.* **58** 977
       Homsy G M 1987 *Ann. Rev. Fluid Mech.* **19** 271
       Couder Y 1991 *Chaos, Order and Patterns* ed R Artuso *et al* (New York: Plenum) pp 203–27

[4]   McCloud K V and Maher J V 1995 *Phys. Rep.* **260** 139
[5]   Bonn D, Kellay H and Meunier J 1998 *Phil. Mag.* B **78** 131
[6]   Bird R B, Armstrong R C and Hassager O 1987 *Fluid Mechanics (Dynamics of Polymeric liquids 1)* (New York: Wiley)
[7]   Bonn D, Kellay H, Bräunlich M, Ben Amar M and Meunier J 1995 *Physica* A **220** 60
[8]   Lindner A, Bonn D and Meunier J submitted
[9]   Bonn D and Meunier J 1997 *Phys. Rev. Lett.* **79** 2662
[10] Rabaud M, Couder Y and Gerard N 1988 *Phys. Rev.* A **37** 935
      Zocchi G, Shaw B E, Libchaber A and Kadanoff L P 1987 *Phys. Rev.* A **36** 1984
[11] Kondic L, Palffy-Muhoray P and Shelley M J 1998 *Phys. Rev. Lett.* **80** 1433
[12] Corvera Poiré E and Ben Amar M 1988 *Phys. Rev. Lett.* **81** 2048
      Ben Amar M 1995 *Phys. Rev.* E **51** 3819
      Ben Amar M and Corvera Poiré E 1999 *Phys. Fluids* **11** 1757
[13] Bonn D, Kellay H, Ben Amar M and Meunier J 1995 *Phys. Rev. Lett.* **75** 2132
[14] Bretherton F P 1961 *J. Fluid Mech.* **10** 166
[15] Ben Amar M, Combescot R and Couder Y 1993 *Phys. Rev. Lett.* **70** 3047